On Altruism, Efficiency and Public Goods

Application: GNU/Linux movement

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(Incomplete version)

Abstract

In this paper we reconsider the problem of altruism and efficiency in an economy with public goods. We present a more complete and natural model of altruism in an two agent economy with public goods than the standard one. We show that welfare comparisons can be made, from the point of view of both, selfish and altruistic agents, regarding selfish vs. altruistic allocations. Given a degree of altruism (or selfishness), we are able to show uniqueness of altruistic equilibria. We also completely characterize the set of altruistic equilibria, for all possible degrees of altruism. We then show that given a degree of altruism or selfishness which produces an inefficient allocation due to market failure, for example, altruistic parameters exist such that the new altruistic equilibrium is Pareto improving from the point of view of both, the original, and the new individuals. We finally illustrate the theory with GNU-Linux, the "open source" movement.

1 Introduction

One issue that has been unresolved in the economic analysis of public goods is the one about welfare comparisons when there is altruism. Two classic papers about altruism and public goods have been Cornes-Sandler (1984, [6]) and Andreoni (1989, [1]). Those papers have introduced the analysis of altruism

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in the economic analysis of public goods. The first deals with ... and the second with issues of efficient government policies when people are altruists. A natural question, though, not addressed by those papers, is if altruism per se can be considered welfare improving when you have market (or other mechanism) implementation problems due to the nature of public goods and asymmetric information (market failure, free riding[1]. Incentive problems with selfish individuals make the provision of public goods generally impossible or inefficient. With altruistic agents things might change and they do, as we show below. On the one hand, the incentive motive to free ride might desapear, at least in part, since people now care about the harmful consequences to other people if they do not report truthfully. On the other hand, depending on the example, altruistic people might just simply organize themselves in order to provide the public good, and the final allocation might be efficient or, at least, Pareto improving with respect to the no provision outcome, or even with respect to the same-price private provision of the public good. In this paper we will focus our attention to the last possibility, since this case is what appears to be happening in some relevant current situations.

In the referred papers, you could ask, once you assumed altruism, efficiency and public policy questions which turn out to be very similar to the ones with selfish individuals. In this paper, on the other hand, we go back and ask if altruism itself could be interpreted as a mechanism, a psychological one, used by selfish agents in order to improve their welfare, given the usual inefficiencies present when selfish people use a given mechanism. In order to answer this question, we have to be precise about different possible levels of altruism. It turns out, as we show below, that not all levels of altruism produce, using the simplest Nash equilibrium solution concept, an efficient outcome.

Our question could be posted as: Given an altruistic equilibrium allocation in an economy with public goods, is it better for selfish individuals to become altruists?. We introduce, in order to be able to make such welfare comparisons, a standard model of altruism, used in macroeconomic models dealing with intergenerational transfers and policy issues[2], as well as in microeconomic models used to explain relatives behavior[3]. Our model also has an interesting relationship, which we make use of, with the central planer

[1] When there is a public good, the market equilibria resulting from the private provision of the good is not Pareto optimal, in general, if the firm can not charge each person a different price. In fact, a Pareto optimal Lindhal equilibrium, in which each person is charged a different price for the public good, requires that each person reveals his or her marginal benefit from enjoying the public good provided. In equilibrium, the personal price equals that marginal benefit. Now, that assumption is not natural in an environment with selfish agents, since they have incentives to understate their interest in the public good, and free ride on the rest of the population.

[2] See, for example the seminal paper of Barro (1974, [3]

[3] See, for example Becker (1974, [4]), Stark (1995, [11]), Arnott and Stiglitz (1991, [12])

characterization of Pareto optimal allocations. It also turns out that the model is a benchmark solution to the problem of consistent interdependent preferences (see Marhuenda and Pérez-Martí [8]), and allows us to consider all possible degrees of altruism, selfishness being a special case of it (altruism of zero degree).

Our model could be thought of as an analysis of the relationship between two central planners. Generalizing in this way welfare analysis, we show that, given an inefficient allocation implemented by any mechanism in an economy populated by individuals with a given profile of altruism, there is another such profile, with a higher degree of altruism, that produces a Pareto improving allocation for both the society with the initial profile, as well as for the society with the more altruistic profile. The mechanism to implement the Pareto improving allocation is the simple Nash equilibrium solution concept.

The plan of the paper is as follows. We first show a couple of results that allow us to make welfare comparisons between people of different degrees of altruism. Then we show existence and uniqueness of Nash equilibrum for an given economy populated by individuals with a given degree of altruism. Finally, we completely characterize the set of altruistic equilibria, and show that the set go "in the right direction" in terms of welfare. A corollary is the result described in the previous paragraph[4]. The uniqueness result, new in the literature of altruism and public goods, enhances the already atractive feature of the Nash equilibrium concept as the simplest mechanism that implements an allocation. As we know, uniqueness is a very useful feature for predictability, and, for that reason, makes the theory easily testable.

In the last section, we show that our model can be interpreted as equivalent to the ones which are standard in the altruism and public goods literature, for example the one by Andreoni ([1, 2]). Since it has more structure, the model is, in fact, a special case. But, as we will show, it allows a more natural interpretation of the concept of altruism in the presence of public goods, and avoids what we consider to be erroneus conceptions in the extreme cases of pure altruism and pure selfishness.

In particular, we are able to use the model in order to explain, from an economic point of view, the recent "open source" software phenomenon, the GNU/Linux movement. We enphasize *economic* point of view, as opposed to a mere "spiritual", or non-sustainable one, since we get that, given a degree of altruism by the Hackers, an efficient equilibrium is quite robust in the same sense any Nash equilibrium is: people do not want to move or deviate from there.

[4] This last result complements another paper of ours ([**?**]), in which we treat the case of incomplete markets, and show that inter-relatives and inter-friends transfers due to altruism are, in fact, a mechanism to "complete the markets", and improve in people's welfare.

Let us examine a little closer this example. Firs of all sofware is a pure public good, since its use by someone does not preclude its use by the rest of the population, even though that use requires apropriate hardware. Among people who has the required hardware, whith enough iddle disk space, a piece of sofware is indeed a pure public, if we assume that its distribution is costless. This is not a farfetched assumption at present, taking into account what is occurring nowadays through the internet. The fact that software is *excludable*, by copyright law does not make it a private good.

Free Software Foundation (FSF), which owns the GNU project, consists of a group of programmers who have spontaneously asociated themselves in order to develop a public access sofware. This institution allows the movement to actually guarantee that the sofware they create is actually a non-excludable ("free access", as they call it) public good. The provision of the good is free of charge, and the person who gets it is asked to voluntarily provide resourses (money, time, machine time, skill) in order to make the project self-sustainable. People who provide resourses, are actually joining the group, since those (*private)*) resourses are an imput in the production of the public good. The *General Public Licence* (GPL) is the legal instrument used by FSF in order to make sure that the person who receives the good is not able to sell it for own profit and/or modify it and distribute it as if that person was the private owner of the good. People can modify it, but only if it is an improvement, or an addition, and the FSF will decide when the modification proposed is appropriate. If it is, the package, when updated, will contain that modification, which will then become free and accesible for all again. The modification or addition is actually part of the private imputs to a new pure, non-excludable, public good, the new version of the piece of software. The "GNU Manifesto", written by Richard Stallman at the beginning of the project, in order to ask for participation and support, says, among other things:

> "I consider that the golden rule requires that if I like a program I must share it with other people who like it. Software sellers want to divide the users and conquer them, making each user agree not to share with others. I refuse to break solidarity with other users in this way. I cannot in good conscience sign a nondisclosure agreement or a software license agreement.... So that I can continue to use computers without dishonor, I have decided to put together a sufficient body of free software so that I will be able to get along without any software that is not free. I have resigned from the AI lab to deny MIT any legal excuse to prevent me from giving GNU away."

In the last section of the paper, Hackers are shown to exhibit altruistic behavior, hiden in their "warm glow" as authors of very sound, free of charge, pieces of software. Even though we do not have a quantitative analysis about the efficiency of their allocations (the quality of their products, basically), we are be able to apply the qualitative conclusions of our paper to this example.

2 The Model

We consider an economy with two agents, one private and one public good. There is no initial endowment of the public good but agent $i = 1, 2$ has ω^i units of initial endowment of the private good. We let $\omega = \omega^1 + \omega^2$ be the total initial endowment of private goods.

Preferences are summarized by the following utility functions $u^1, u^2 : \mathbb{R}^2_+ \times \mathbb{R}_+ \to \mathbb{R}$

$$\begin{aligned} u^1(x_1, x_2, y) &= v^1(x_1, y) + \gamma_1 v^2(x_2, y) \\ u^2(x_1, x_2, y) &= v^2(x_2, y) + \gamma_2 v^1(x_1, y), \end{aligned} \tag{1}$$

where $0 \leq \gamma_1, \gamma_2$. Here, x_i represents the consumption of the private good by individual $i, i = 1, 2$, and y is the output (and consumption) of the pure public good. Selfish preferences correspond to $\gamma_1 = \gamma_2 = 0$ and a high degree of altruism is associated with large values of γ_1 or γ_2. Let the function $f : \mathbb{R} \to \mathbb{R}$, be the technology available in society that transforms private goods into public goods.

Assumption 2.1 *Public good technology is represented by a mapping $f : \mathbb{R}_+ \to \mathbb{R}_+$, that is a strictly increasing, strictly concave C^1 function, and $f(0) = 0$.*

Thus, an economy is specified by a vector $(v^1, v^2; f; \gamma_1, \gamma_2; w^1, w^2)$, consisting of the preferences discussed above, and the initial endowment of the single private good w^i available to agent $i = 1, 2$. The resource constraints for the economy are, then: $x_1 + x_2 + z = w^1 + w^2 = \omega$, and $y = f(z)$, where z is the amount of input of private goods used for the production of the public good.

An allocation (x_1, x_2) is feasible if $0 \leq x_i \leq \omega^i$. It is understood here that x_i is the consumption by agent $i = 1, 2$ of the private good and that an amount $f(\omega - x_1 - x_2)$ of the public good is provided along with the allocation (x_1, x_2). It is convenient to substitute for the value of y in preferences. We make this more precise in the following

Assumption 2.2 *The preferences of the agents are represented by utility functions*

$$\begin{aligned} W^1(x_1, x_2; \gamma_1) &= v^1(x_1, f(\omega - x_1 - x_2)) + \gamma_1 v^2(x_2, f(\omega - x_1 - x_2)) \\ W^2(x_1, x_2; \gamma_2) &= v^2(x_2, f(\omega - x_1 - x_2)) + \gamma_2 v^1(x_1, f(\omega - x_1 - x_2)) \end{aligned}$$

where $\gamma_1, \gamma_2 \geq 0$ and v^i are strictly concave functions strictly increasing in both arguments and such that for $i \neq j$,

$$v_{ij}^k = \frac{\partial^2 v^k}{\partial x_i \partial x_j}(x_1, x_2) \geq 0 \tag{2}$$

and

$$\lim_{x_k \to 0} \frac{\partial v^k}{\partial x_k}(x_1, x_2) = +\infty. \tag{3}$$

Definition 2.3 *A feasible allocation (x_1, x_2) is Pareto Optimal if there is no other feasible allocation (r_1, r_2) such that $W^i(r_1, r_2; \gamma_i) \geq W^i(x_1, x_2; \gamma_i)$ with some inequality strict. We let $\mathrm{P}(\gamma_1, \gamma_2)$ denote the set of Pareto optimal allocations in the economy (γ_1, γ_2).*

We represent in the following graph feasible allocations by points (x^1, x^2) such that $\omega - x^1 + x^2 \geq 0$. y then can be inferred as $y = f(\omega - x^1 + x^2 \geq 0)z \in \mathbb{R}_+\}$. We use a triangle similar to Kolm's to represent allocations, preferences, Pareto optimal allocations and equilibria:

Next result allows us to make preference comparisons. It can be viewed as a more general approach to central planner welfare analysis.

Proposition 2.4 *Consider vectors $\gamma = (\gamma', \gamma'') \geq 0$ such that $\gamma'\gamma'' \leq 1$. Then the Pareto correspondence P is decreasing in γ, i.e. if $(\gamma_1, \gamma_2) \geq (\gamma_3, \gamma_4)$, then $\mathrm{P}(\gamma_1, \gamma_2) \subset \mathrm{P}(\gamma_3, \gamma_4)$.*

Proof: The Proposition is trivial if $\gamma_1 = \gamma_2 = 0$. Let us assume, for example, that $\gamma_2, \gamma_4 > 0$. An allocation (x_1, x_2) is Pareto optimal in the economy $\gamma = (\gamma_1, \gamma_2)$ iff there is an $\alpha \in [0, 1]$ such that (x_1, x_2) maximizes

$$\begin{aligned} & \alpha W^1(x_1, x_2; \gamma_1) + \tfrac{(1-\alpha)}{\gamma_2} W^2(x_1, x_2; \gamma_2) \\ = \; & v^1(x_1, f(\omega - x_1 - x_2)) + (\alpha\gamma_1 + \tfrac{(1-\alpha)}{\gamma_2}) v^2(x_2, f(\omega - x_1 - x_2)). \end{aligned}$$

Let

$$\beta = \frac{\alpha\gamma_1 + (1-\alpha)/\gamma_2 - 1/\gamma_4}{\gamma_3 - 1/\gamma_4}.$$

Since $\gamma_4 \leq \gamma_2$ and $\gamma_1\gamma_2 \leq 1$, we have

$$\alpha\gamma_1\gamma_2\gamma_4 + (1-\alpha)\gamma_4 \leq \alpha\gamma_4 + (1-\alpha)\gamma_4 = \gamma_4 \leq \gamma_2$$

so $\alpha\gamma_1 + (1-\alpha)/\gamma_2 - 1/\gamma_4 \leq 0$. Clearly, $\gamma_3 - 1/\gamma_4 < 0$, since $\gamma_3\gamma_4 \leq 1$. Hence $\beta > 0$. On the other hand,

$$\alpha + \frac{1-\alpha}{\gamma_1\gamma_2} \geq 1 \geq \frac{\gamma_3}{\gamma_1}$$

so $\alpha\gamma_1 + (1-\alpha)/\gamma_2 - 1/\gamma_4 \geq \gamma_3 - 1/\gamma_4$ and we also have $\beta \leq 1$.

Finally, we see that $\alpha W^1(x_1, x_2; \gamma_1) + \frac{(1-\alpha)}{\gamma_2} W^2(x_1, x_2; \gamma_2) = \beta W^1(x_1, x_2; \gamma_3) + \frac{(1-\beta)}{\gamma_4} W^2(x_1, x_2; \gamma_4)$ so (x_1, x_2) is Pareto optimal in the economy $\gamma = (\gamma_3, \gamma_4)$ as well. **Q.E.D.**

Even tough we have the possibility of welfare comparisons now, there is a slight complication. Someone might argue that even though an increase in the altruistic profile of a society keeps some of the previous efficient allocations still efficient, society might be losing some key efficient allocations. Next result shows that it is not the case.

Proposition 2.5 *Consider vectors $\gamma = (\gamma', \gamma'') \geq 0$ such that $\gamma'\gamma'' \leq 1$. Let (x_1, x_2) be an inefficient allocation for society (γ_1, γ_2), and let $\mathrm{P}(\gamma_1, \gamma_2; x_1, x_2)$ be the Pareto efficient allocations preferred by both individuals to (x_1, x_2) Then there exists an economy $(\gamma_3, \gamma_4) \geq (\gamma_1, \gamma_2)$ such that $\mathrm{P}(\gamma_1, \gamma_2; x_1, x_2) \subset \mathrm{P}(\gamma_3, \gamma_4)$.*

Proof: To be provided.

Definition 2.6 A feasible allocation (x_1, x_2) is an Altruistic Agreement if $\gamma_1\gamma_2 \leq 1$, x_1 is a solution to

$$\max_{0 \leq \xi \leq w^1} W^1(\xi, x_2; \gamma_1) \tag{4}$$

and x_2 is a solution to

$$\max_{0 \leq \xi \leq w^2} W^2(x_1, \xi; \gamma_2). \tag{5}$$

This is the application of the Nash equilibrium concept to our model. It turns out that, in general, there is an inherent conflict among the agents of this economy, in spite of their altruism. Only in the case $\gamma_1\gamma_2 = 1$, what each agent wants for himself of the private good coincide exactly with what the other agent wants for him (the peaks for equations ?? coincide for both agents), and the preferred quality of the public good also coincide[5]. For the other cases, we have to use the Nash equilibrium concept in order to guarantee that they agree upon a common quality of the (common) public good. Now, in the case $\gamma_1\gamma_2 < 1$, Nash equilibrium entails the kind of disagreement that says that each agent, if he were to choose how to allocate the endowments of all the economy, he would prefer to give less of his own private endowment for the production of the public good, and more of the private endowment of the other individual for that purpose. In the case $\gamma_1\gamma_2 > 1$, a Nash equilibrium would prescribe an allocation such that, if each agent were to do a similar choice, he would prefer to give more to the public good, and the other agent to give less, of their own resources, than the amounts prescribed in the proposed equilibrium. As we will see, the case $\gamma_1\gamma_2 = 1$ generates an equilibrium that is a Pareto allocation for both economies, the one with selfish, as well as the one with altruistic people. Furthermore, the Pareto allocation for the altruistic economy is, in that case, the unique possible Pareto allocation.

Proposition 2.7 *Given $\gamma_1, \gamma_2 \geq 0$, $\gamma_1\gamma_2 \leq 1$, there is (at least) an Altruistic Agreement in the economy (γ_1, γ_2).*

Proof: It is easy to verify that the mappings W^i are strictly concave and thus, given $\theta \in [0, w^2]$, there is a unique solution $\varphi_1(\theta, \gamma_1) \in [0, w^1]$ to the maximization problem $\max_{0 \leq \xi \leq w^1} W^1(\xi, \theta; \gamma_1)$. This defines a mapping $\varphi_1 : [0, \omega^2] \to [0, \omega^1]$ representing the best response (in terms of consumption of the private good) of player 1 given that agent 2 consumes θ. By the Theorem of the Maximum, the function φ_1 is continuous. Similarly, there is a continuous mapping $\varphi_2 : [0, \omega^1] \to [0, \omega^2]$ such that $\varphi_2(\theta, \gamma_2) \in [0, w^2]$ is the unique solution to the maximization problem $\max_{0 \leq \xi \leq w^2} W^2(\theta, \xi; \gamma_2)$. Note that (x_1, x_2) is a Altruistic Agreement if and only if $\varphi_1(x_2; \gamma_1) = x_1$ and $\varphi_2(x_1, \gamma_2) = x_2$.

Now, the function $\varphi_2 \circ \varphi_1 : [0, \omega^2] \to [0, \omega^2]$ is also continuous. By the fixed point theorem, there is $\theta \in [0, \omega^2]$ such that $\varphi_2 \circ \varphi_1(\theta, \gamma_1) = \theta$. Thus, $x_1 = \varphi_1(\theta, \gamma_1)$ and $x_2 = \theta$ is a Altruistic Agreement. **Q.E.D.**

[5] In fact, if the maximization problems (4) and (5) were performed over the two first arguments, an internal single peak vector would be obtained as solution, given our assumptions. The results of Marhuenda and Pérez-Martí ([**?**]) that says that the peaks coincide when $\gamma_1\gamma_2 = 1$ apply here too.

We make the necessary assumptions, so that the solutions the the problems 4 and 5 are determined by the first order conditions. In fact, the mappings φ_1 and φ_2 introduced in the proof of Proposition 2.7 are defined implicitly by the equations $D_1 W^1(\varphi_1(\theta,\gamma_1),\theta;\gamma_1) = 0$ and $D_2 W^2(\theta,\varphi_2(\theta,\gamma_2);\gamma_2) = 0$ and so are C^1. Note that φ_1 and φ_2 depend also on γ_1 and γ_2, respectively.

Lemma 2.8 *1. For γ_1,γ_2 fixed, φ_i, $i = 1,2$ is strictly decreasing in θ.*

2. For θ fixed, φ_1 is strictly decreasing in γ_1, φ_2 is strictly decreasing in γ_2.

Proof: By implicit differentiation, we have that $\varphi_1' = \partial\varphi_1/\partial\theta$ is derived from equation $D_{11}W^1(\varphi_1(\theta,\gamma_1),\theta)\varphi_1'(\theta) + D_{12}W^1(\varphi_1(\theta,\gamma_1),\theta) = 0$. Thus,

$$\varphi_1'(\theta,\gamma_1) = -\frac{D_{12}W^1(\varphi_1(\theta,\gamma_1),\theta)}{D_{11}W^1(\varphi_1(\theta,\gamma_1),\theta)}.$$

A simple computation shows (using assumptions 2.1 and 2.2) that

$$D_{11}W^1 = v_{11}^1 - 2v_{12}^1 f' + v_{22}^1(f')^2 + v_2^1 f'' + \gamma_1(v_{22}^2(f')^2 + v_2^2 f'') < 0$$

and

$$D_{12}W^1 = -v_{12}^1 f' + v_{22}^1(f')^2 + v_2^1 f'' - \gamma_1(v_{12}^2 f' - v_{22}^2(f')^2 - v_2^2 f'') < 0$$

so $\varphi_1'(\theta,\gamma_1) < 0$ and φ_1 is decreasing in θ.

By differentiating now implicitly $D_1W^1(\varphi_1(\theta,\gamma_1),\theta;\gamma_1) = 0$ with respect to γ_1 we have

$$D_{11}W^1\frac{\partial\varphi_1}{\partial\gamma_1} + \frac{\partial}{\partial\gamma_1}D_1W^1 = 0$$

which is the same as

$$D_{11}W^1\frac{\partial\varphi_1}{\partial\gamma_1} = v_2^2 f'$$

But, $D_{11}W^1 < 0$ and $v_2^2 f' > 0$, so $\frac{\partial\varphi_1}{\partial\gamma_1} < 0$. A similar argument applies to φ_2. **Q.E.D.**

Proposition 2.9 *Given $\gamma_1,\gamma_2 > 0$, $\gamma_1\gamma_2 \leq 1$, the Altruistic Agreement of Proposition 2.7 is unique.*

Proof: By Lemma 2.8, for each γ_1 fixed, the function φ_1 is monotonic in θ, and thus, it has an inverse $\psi_1 = \varphi_1^{-1}$. An altruistic Agreement is determined by a point θ such that $\psi_1(\theta,\gamma_1) = \varphi_2(\theta,\gamma_2)$.

We claim now that $\psi_1'(\theta,\gamma_1) < \varphi_2'(\theta,\gamma_2)$ for $0 < \gamma_1\gamma_2 \leq 1$ and all θ in the domain of ψ_1. To see this fix γ_1, γ_2 such that $0 < \gamma_1\gamma_2 \leq 1$. Let $\gamma_1' = 1/\gamma_2$. Note that $\gamma_1 \leq \gamma_1'$. After some computation we have that

$$\begin{aligned}
\varphi_2'(\theta,\gamma_2) &= -\frac{D_{12}W^2(\varphi_1(\theta,\gamma_1),\theta)}{D_{22}W^2(\varphi_1(\theta,\gamma_1),\theta)} \\
&= \frac{v_{12}^2 f' - v_{22}^2(f')^2 - v_2^2 f'' + \gamma_2(v_{12}^1 f' - v_{22}^1(f')^2 - v_2^1 f'')}{v_{11}^2 - 2v_{12}^2 f' + v_{22}^2(f')^2 + v_2^2 f'' + \gamma_2(v_{22}^1(f')^2 + v_2^1 f'')}
\end{aligned}$$

Since W^1 is strictly concave, the determinant

$$\begin{vmatrix} D_{11}W^1 & D_{12}W^1 \\ D_{12}W^1 & D_{22}W^1 \end{vmatrix} = D_{11}W^1 D_{22}W^1 - (D_{12}W^1)^2$$

is strictly positive. Thus,

$$\begin{aligned}
\psi_1'(\theta,\gamma_1) &= -\frac{D_{11}W^1(\varphi_1(\theta,\gamma_1),\theta)}{D_{12}W^1(\varphi_1(\theta,\gamma_1),\theta)} \\
&< -\frac{D_{12}W^1(\varphi_1(\theta,\gamma_1),\theta)}{D_{22}W^1(\varphi_1(\theta,\gamma_1),\theta)} \\
&= -\frac{-v_{12}^1 f' + v_{22}^1(f')^2 + v_2^1 f'' - \gamma_1(v_{12}^2 f' - v_{22}^2(f')^2 - v_2^2 f'')}{v_{22}^1(f')^2 + v_2^1 f'' + \gamma_1(v_{11}^2 - 2v_{12}^2 f' + v_{22}^2(f')^2 + v_2^2 f'')} \\
&< \frac{v_{12}^1 f' - v_{22}^1(f')^2 - v_2^1 f'' + \gamma_1'(v_{12}^2 f' - v_{22}^2(f')^2 - v_2^2 f'')}{v_{22}^1(f')^2 + v_2^1 f'' + \gamma_1'(v_{11}^2 - 2v_{12}^2 f' + v_{22}^2(f')^2 + v_2^2 f'')} \\
&= \frac{v_{12}^2 f' - v_{22}^2(f')^2 - v_2^2 f'' + \gamma_2(v_{12}^1 f' - v_{22}^1(f')^2 - v_2^1 f'')}{v_{11}^2 - 2v_{12}^2 f' + v_{22}^2(f')^2 + v_2^2 f'' + \gamma_2(v_{22}^1(f')^2 + v_2^1 f'')} \\
&= \varphi_2'(\theta,\gamma_2)
\end{aligned}$$

where the last inequality follows because $v_{12}^2 f' - v_{22}^2(f')^2 - v_2^2 f'' > 0$, $v_{11}^2 - 2v_{12}^2 f' + v_{22}^2(f')^2 + v_2^2 f'' < 0$ and $\gamma_1 \leq \gamma_1'$. The claim is now proved.

Suppose now that there are two Altruistic Agreements $\theta_1 < \theta_2$ such that $\psi_1(\theta_1) = \varphi_2(\theta_1)$ and $\psi_1(\theta_2) = \varphi_2(\theta_2)$. Then $\psi_1(\theta) - \varphi_2(\theta,\gamma_2)$ has two zeros in the interval $[\theta_1,\theta_2]$, so there is $\theta_3 \in (\theta_1,\theta_2)$ such that $\psi_1'(\theta_3) = \varphi_2'(\theta_3)$ which contradicts the claim. **Q.E.D.**

In the following graph, in which the Pareto frontier for selfish agents is shown, we illustrate two Altruistic Agreements as the crossings of the reaction curves ψ and φ for different gamma vectors. There, $\psi' < \varphi < 0$. In the case $\gamma_1\gamma_2 = 1$, $\psi(\ ,\gamma_1)$ and $\varphi(\ ,\gamma_2)$ cross exactly at the Pareto frontier of the selfish agents ($P(0,0)$), and $\mathrm{P}(\gamma_1,\gamma_2)$ consists of just that point, which is the unique Altruistic Agreement. We have also represented the case of (γ_1,γ_2'), with $\gamma_2' < \gamma_2$), so that $\gamma_1\gamma_2' < 1$. Here $\psi(\ ,\gamma_1)$ and $\varphi(\ ,\gamma_2')$ cross at an inefficient allocation, above $P(0,0)$. In this case, $P(\gamma_1, \gamma_2')$ is the segment of $P(0,0)$ that goes from the point where $\varphi(\ ,\gamma_2')$ crosses it to $\mathrm{P}(\gamma_1, \gamma_2)$.

In order to fully characterize the set of all possible Altruistic Agreements (agreements for all possible gamma combinations), we analyze the structure of the mapping

$$(\gamma_1, \gamma_2) \mapsto \mathrm{R}(\gamma_1, \gamma_2).$$

It assigns to every economy (γ_1, γ_2) such that $\gamma_1, \gamma_2 > 0$ $\gamma_1\gamma_2 \leq 1$ its unique Altruistic Agreement. First note that $\mathrm{R}(\gamma_1, \gamma_2)$ is given implicitly by the system of equations $D_1W^1 = D_2W^2 = 0$. The Jacobian is $D_{11}W^1D_{22}W^2 - D_{12}W^1D_{12}W^2 > 0$ if $\gamma_1, \gamma_2 > 0$. Thus, $\mathrm{R}(\gamma_1, \gamma_2)$ is a C^1 function of (γ_1, γ_2). By Proposition 2.8, the function R defines a diffeomorphism from the open set $A = \{(\gamma_1, \gamma_2) \in \mathbb{R}^2 : \gamma_1, \gamma_2 > 0, \gamma_1\gamma_2 < 1\}$ into the set of feasible allocations which extends to the boundary $\partial A = \{(\gamma_1, \gamma_2) \in \mathbb{R}^2_+ : \gamma_1\gamma_2 = 0\} \cup \{(\gamma_1, \gamma_2) \in \mathbb{R}^2_+ : \gamma_1\gamma_2 = 1\}$. Let $B = \{(\gamma_1, \gamma_2) \in \mathbb{R}^2_+ : \gamma_1\gamma_2 = 1\}$.

Proposition 2.10 $\mathrm{P}(0,0) = \mathrm{R}(B) \cup \{(\bar{x}_1, 0)\} \cup \{(0, \bar{x}_2)\}$, *where, for* $i = 1, 2$, $\bar{x}_i$ *maximizes*

$$\max_{0 \leq z \leq w} v_i(z, f(\omega - z)$$

.

It follows that $\mathrm{R}(A)$ is a region of $\mathbb{R}^2$ with a non–empty interior whose boundary is determined by $\mathrm{P}(0,0)$ and the graphs of the mappings $\varphi_1(\theta, 0), \varphi_2(\theta, 0)$. $\mathrm{R}(A)$ can be seen in the following graph. In general, a market equilibrium under-produces public goods, and generate allocations above $P(0,0)$. A straightforward consequence of the characterization of $\mathrm{R}(A)$ and of Proposition 2.5 is that, given a market allocation of public goods which is inefficient, there are altruistic parameters that improves upon that allocation in the Pareto sense. Formally:

Corollary 2.11 *Consider vectors* $\gamma = (\gamma', \gamma'') \geq 0$ *such that* $\gamma'\gamma'' \leq 1$. *Let* (x_1, x_2) *be an inefficient allocation for society* (γ_1, γ_2). *Then there exists an economy* $(\gamma_3, \gamma_4) \geq (\gamma_1, \gamma_2)$ *such that, if* (x_3, x_4) *is the Altruistic Equilibrium that corresponds to economy* (γ_3, γ_4), *this allocation is Pareto preffered to* (x_1, x_2) *in both economies.*

3 Example: Efficiency of "warm glow" Giving

In our original model, preferences for person 1 where given by utility function

$$u^1(x_1, x_2, y) = v^1(x_1, y) + \gamma_1 v^2(x_2, y) \tag{6}$$

Let $z_i = w^i - x_i, i = 1, 2$. So z_i is the amount of private good that person i contributes to the provision of the public good. If person 1 has enough knowledge to realize that $y = f(z)$, where $z = z_1 + z_2$, then, since $y = f(z_1 + z_2) = f(z_1 + w^2 - x_2)$, and f is invertible, he can figure that $x_2 = w^2 + z_1 - f^{-1}(y)$. Having this relations in mind, person 1 realizes that, from (6),

$$\begin{aligned} u^1(x_1, x_2, y) &= v^1(x_1, y) + \gamma_1 v^2(x_2, y) \\ &= v^1(x_1, y) + \gamma_1 v^2(w^2 + z_1 - y, y) \\ &\equiv G^1(x_1, y, z_1), \end{aligned} \tag{7}$$

which is an equivalent formulation to the one of Andreoni ([2]). Now, in Andreoni's formulation, people do not seem to realize directly that their amount of gifts to the public good affect the level of private consumption of others. Our model provides an interpretation regarding where the "warm glow" might come from: it certainly has to do with the relationship of the person in question with others[6]. In our model, the warm glow comes from the satisfying fact that own gifts to public good enhance others people's pleasure of consuming both, private and public kind of goods. In fact, given an amount of public good, an increase in own gifts increases consumption of the private goods by others, since your labor, say, is a substitute for the labor of others. On the other hand, given an amount of consumption of the private good by others, an increase in own gift to public good increases everybody's consumption of the public good.

4 Final remarks

We have shown that, indeed, we can interpret altruism in an economy with public goods as a mechanism people use in order to improve their welfare. We have not talked about the evolution process that might generate changes in

[6] After all, our main motivation is why people are altruists in the first place. Even though we do not show the process of learning or evolution of altruistic preferences here, the paper argues that a justification for altruism that it makes (selfish or altruist) people better off under certain circumstances.

preferences towards altruism. Our models shows, as it should be obvious, that if there a fitness function, it should be referred to society, and not necessarily to individuals. In fact, they have an incentive to deviate and stay selfish if they know the other members of society are going to evolve towards altruism. Now, societies with a right enough profile of altruism might be more successful than others due to the fact that they are more efficient in the absence of a mechanism that implements efficient outcomes for the selfish society members. A certain degree of reciprocity might be achieved, to complement society fitness, if there is a certain threshold in consumption below which individuals desapear from society and weakens society economic potential.

We want to pursue this line of research, extending, at the same time, the size of society to any finite natural number $n > 2$. In order to extend the number of society members, a complication shows up: if we require that individuals care for every other person in society, the assumed level of rationality regarding amount of information and computational ability becomes huge and not realistic. The model would have the same criticism of impracticality of the central planner solutions, only that this time the criticism would have to be multiplied, at least, by the number n. To keep things simple, we are working on an extension that requires that any individual knows and cares only about his neighbor. It turns out that evolution towards altruism has been shown[7] to be robust only in the cases of complete information. Individualistic preferences become evolutively robust in presence of incomplete information. Our requirement for extension of the model has, then, two advantages: it is realistic (people know and care, at least, about their neighbors), and it makes the model evolutionarily stable.

We make now some remarks comparing definitions of altruism. We think our definition is more appropriate than Andreoni's, the standard one in the literature of altruism and public goods. For Andreoni, when G^1 does not depend on z_1, person 1 is a "pure altruist". In our model, this case corresponds to having $\gamma_1 = 0$, the case when the person is a "pure selfish" individual. The rationale Andreoni presents for this is that when one cares about the public good one consumes, one is caring (indirectly) about the others who consume it. But that is not a necessity, as the related case of externalities shows. In our model we see that the fact that an individual cares about public goods (when y appears in the "pleasure" function $v(x, y)$) doesn't really mean that he is an altruist in the sense of caring for others. Indeed, even if he does not provide anything to its production (implying that only other persons are experiencing desutility because of their effort), he still enjoys consuming it.

Similarly, when G^1 does not depend on y, person 1 is a "pure selfish" individual for Andreoni. Our interpretations coincide when individual 1 does

[7] See Ok and Vega-Redondo (1999, [10]).

not care for the public good and individual 2 does, but $\gamma_1 = 0$. Now, in our model, this might happen when both agents do not care for the public good, even though agent 1 is an altruist. This is not a relevant case for an economy with public goods, though. But our model allows for a more insightful case: when individual 1 does not care for the public good and individual 2 does, and $\gamma_1 > 0$. In this case, the more of the private good person 1 gives away, the more person 2 benefits. Now, the action of giving away resources lowers person 1 consumption, of course. So person 1 is improving person 2's welfare at his own expense. Can this be called pure selfishness?. If anything this is pure altruism.

In this last case, Andreoni's interpretation comes from considering that if one cares for what one gives, directly, what one is caring about is not what the others get from it, but about the fact that "one feels good" about it. We disagree with this interpretation, since there might be a reason, explicit or not, for that feeling. Indeed, one of the purposes of our paper is to show that "true altruism", which means caring about others, has probably to do with selfish individuals looking for a way of improving their own welfare in the presence of a market failure, say. As we said above, our model of altruism also gives an interpretation about where the warm glow might come from, and that interpretation makes sense. Now, Andreoni's results apply to what he really thinks happens in reality, the case of "impure altruism", our normal altruism case. So, both interpretations might be said to coincide in all the cases, except in the extreme cases, where ours is more natural. In this sense, the extra structure of our model also has a payoff .

We finally remark that the warm glow function G allows us to include in our model the case of software Hackers. By Hackers we mean not only the expert Hackers, in the usual sense, but also the many volunteers who provide resources in any amount for the common good (like reporting bugs). Apart from the fact that the model stresses the possible economic robustness of the open source movement, the paper opens a window to consider in a new light the copyright laws. It might be inefficient to enforce a copyright law implying exclusion of a public good when it has been produced by a spontaneous altruistic movement[8]. So property rights of law-excludable public goods should be reconsidered in at least some cases, in light of efficiency considerations. Also, public policy considerations regarding economic growth due to technological development might include the alternative of preference formation and support to altruistic groups that produce such law-excludable public goods.

[8]Think of the case of UNIX, produced originally by what we should well consider an altruistic group, appropriated by some other people, retarding the Linux phenomenon.

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